The Effect of Instruction on Students' Generation of Diagrams

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Although the use of diagrams is advocated in mathematics, support for this instructional practice appears to be intuitive rather than evidentiary. A case study was used to evaluate the effectiveness of instruction in diagram generation with Year 5 students. The results suggest that although instruction can have a positive effect on students' diagram generation, the success of the program is dependent on the teachers' understanding of the role of diagram generation in problem solving and how diagram generation can be facilitated.

Using Diagrams in Problem Solving

Although the strategy *draw a diagram* is advocated in mathematical problem solving (e.g., Australian Education Council, 1991) for its assumed cognitive advantages (e.g., Nunokawa, 1994; van Essen & Hamaker, 1990), empirical justification for the use of the strategy *draw a diagram* is equivocal (Shigematsu & Sowder, 1994; Simon, 1986). Shigematsu and Sowder (1994) have asserted the need for more research and theory on this topic to validate and inform instructional practice. Visual literacy is a long neglected field (Balchin & Coleman, 1965; Box & Cochenour, 1994). Thus, the effective use of a diagram is a significant and urgent problem for mathematics education research.

The use of the diagram in problem solving assumes two roles: (a) as a representation of the *problem structure,* and (b) as the basis for the development of a *solution structure* (Carroll, Thomas, Miller, & Friedman, 1980). The problem structure is comprised of the relationships among the problem information and the presentation of that information, whereas the solution structure is comprised of the solution process and the solution (Carroll et al, 1980). As the solution structure is dependent on the solver's perception of the problem structure, the generation of an appropriate diagram is integral to a successful solution (Yancey, Thompson, & Yancey, 1989): "Generating a drawing does not guarantee that one finds the correct solution, but merely increases the chance that a problem will be conceptualized correctly" (van Essen & Hamaker, 1990, p. 309). Henceforth, the focus of this paper is on the generation of diagrams in problem solving.

As the correspondence between the diagram and the problem structure determines the value of a diagram for problem solving (Veloo & Lopez-Real, 1994), the diagrams generated by students should represent the structural features of the problem. Whereas novices base their representations on the literal features of the problem, experts incorporate relevant background knowledge into their representations (Chi, Feltovich, & Glaser, 1981). Notwithstanding the appropriateness of the diagrams which are generated, an additional concern is that students are reluctant to *draw a diagram* even when specifically directed (Veloo & Lopez-Real, 1994) and even consider the use of a diagram to be inappropriate (Shigematsu & Sowder, 1994).

Instruction in Diagram Generation

However, when instruction has been implemented the results have been disappointing (e.g., Simon, 1986), not unexpectedly, because of defects in: (a) the content of the instructional program, (b) the selection of tasks for evaluation, and (c) the evaluation criteria.

The content of an instructional program should focus on using diagram drawing to enhance conceptualisation because the advantage of generating a diagram relates to the conceptualisation of the problem (van Essen & Hamaker, 1990). Simon's (1986) instructional program comprised six diagram drawing subskills. Of her subskills, only (1) representing all relevant information, and (2) creating an integrated diagram are critical to the conceptualisation of the problem. In contrast, the subskills of (3) labelling completely,

(4) checking the accuracy of the diagram, and (5) drawing multiple representations are not critical. Indeed the final subskill of (6) verbalising what is represented and what needs to be represented may inhibit problem solving (Schooler, Ohlsson, & Brooks, 1993). Furthermore, an instructional program should also include the different types of diagrams that have unique problem structures: networks (ie., path or line diagrams), hierarchies, matrices, and a range of diagrams that exhibit part-whole characteristics (Novick & Francis, 1993).

The selection of tasks for evaluation was problematic in Simon's (1986) study as they do not appear to have been sufficiently challenging as indicated by high pretest scores and the acknowledged ceiling effect. Novel problems rather than routine problems should be used for evaluation because in novel problems the students need to derive the problem structure rather than simply implement the solution structure.

The evaluation criteria used by Simon (1986) were: (1) the type of diagram, (2) the accuracy of the diagram, (3) the completeness of the diagram, and (4) the labelling of the diagram. The type of diagram generated and the accuracy of the diagram are appropriate criteria because they are integral to the recognition and representation of the problem structure (Novick, 1996). However labelling is inappropriate because although the diagram may have been used as a cognitive tool, the student may have had no communicative intent. Completeness is also inappropriate because as the student conceptualises the problem his or her internalized representations may be more developed than his or her external representation (Hegarty & Narayanan, 1994). Frequency of diagram generation is an additional appropriate criteria because students are reluctant to use diagrams (e.g., Veloo & Lopez-Real, 1994). Furthermore, the extent of a student's autonomy in diagram generation is also an important criteria because there may be a change in a student's *Zone of Proximal Development* which can be discerned through scaffolding (Vygotsky, 1978). Scaffolding can be used to ascertain how much teacher support is required to assist the student to generate a diagram. Although the type of diagram, frequency of diagram generation, and autonomy of diagram generation can be readily established, ascertaining the accuracy of a diagram is problematic. As a problem structure can be represented diagrammatically in various ways, there is no single "correct" diagram. However as prototypes are useful for ascertaining expertise when the degree of similarity between the exemplars (i.e., diagrams) may be low (Sternberg & Horvarth, 1995), a *theoretical prototype* can be developed by applying the properties of the type of diagram (Novick, 1996) to the context of the problem. The accuracy of the diagram can then be established by comparing the congruence between the student's diagram and the prototype.

Method

A case study design (Yin, 1994) was used to investigate the effect of instruction on the *generation of diagrams,* which is one aspect of ongoing evaluative research on diagram use in problem solving (Diezmann, 1995; 1996). It was hypothesised that there would be an improvement in students' diagram generation as a consequence of an instructional program consisting of twelve half-hour lessons. In order to optimise the conditions for success, each of the issues discussed earlier in relation to Simon's (1986) study was accommodated: (a) *the content of the instructional program,* (b) *the selection of tasks for evaluation,* and (c) *the evaluation criteria.*

The informants in the study were 12 Year 5 students with a mean age of 10 years 3 months from a moderately sized parochial school in a lower socio-economic suburb in Brisbane. Tasks comprising isomorphic sets of five novel problems were presented to each informant during 30 minute interviews conducted before and after instruction. As the informants were not specifically instructed to use a diagram, those informants who did not spontaneously use a diagram were given further opportunities to generate a diagram through scaffolding in the form of *general prompts* (e.g., Is there any other way you could do the problem?) or *specific prompts* (e.g., Could you draw a diagram or picture?). Specific prompts were used to investigate whether the informants: (a) didn't think to generate a diagram, or (b) couldn't generate a diagram. The interviewer was known to the subjects through prior classroom involvement and the interviews were video-taped and subsequently transcribed. *The Koala and The Frog* tasks (see Figure 1) are reported because there were distinctive visual differences between the diagrams generated for the pre- and post-instruction interviews respectively.

The Koala (Pre-instruction)	**The Frog (Post-instruction)**
A sleepy koala wants to climb to the top of a gum tree that is 10 metres high. Each day the koala climbs up 5 metres, but each night, while asleep, slides back 4 metres. At this rate how many days will it take the koala to reach the top?	A frog was trying to jump out of a well. Each time the frog jumped, it went up four rows of bricks, but because the bricks were slippery it slipped back one row. How many jumps will the frog need to make if the well is 22 rows high?

Figure 1. The Koala and Frog Tasks

Results and Discussion

The implementation of this study led to two assertions.

Assertion 1: After instruction, there will be a positive quantitative change in the diagrams that are generated independently. After instruction, the number of informants who spontaneously generated a diagram increased from four to eleven. However providing informants with verbal prompts also facilitated the generation of a diagram. Although no informants responded to a general prompt on the pre-instruction task by drawing a diagram, six of the eight informants who did not draw a diagram responded to the specific prompt. Three of these informants spontaneously generated a diagram, and the other three informants appeared uncertain but generated a diagram upon request. Their uncertainty appeared to be related to either a difficulty representing the problem structure or the perceived utility of the diagram. For example, Jon stated "I thought of drawing a 10 metre high, a 10 metre tree. I don't have enough room to do a 10 metre tree" and Candice shrugged her shoulders when asked: "Would it help (to draw a diagram)?" Two informants responded negatively to the specific prompt. On the post-instruction task, Adrian was the only informant who did not generate a diagram spontaneously. However he responded to a general prompt with the statement "By a line (diagram or network)", which suggests that although he did not generate a diagram, he at least knew the correct type of diagram to draw. This response was a stark contrast to his head shaking on the pre-instruction task when given a similar prompt.

As shown on Table 1, after instruction there was a positive quantitative change in: (a) the number of diagrams drawn, and (b) the degree of scaffolding required to produce a diagram or identify the type of diagram to be drawn. No negative quantitative differences were evident. However despite the quantitative improvement in diagram generation not all of these diagrams were accurate representations of the problem structure.

Table 1
Spontaneous Generation of a Diagram (*n*=12)

degree of scaffolding required to generate a diagram	*The Koala* task (pre-instruction)	*The Frog* task (post-instruction)
no scaffolding	4	11
a general prompt	0	1
a specific prompt	6	0

Assertion 2: After instruction, there will be a positive qualitative change in the diagrams that are generated independently. The appropriate type of diagram for these problems was a network (Novick, 1996). Although all diagrams generated by students could be classified as networks, some diagrams were structurally inaccurate when compared to the theoretical prototype. The prototype consisted of *nodes* depicting either the metre marks or the bricks, depending on the problem, and a depiction of *movement* between the nodes.

On the pre-instruction task eight of the twelve informants represented the *starting and finishing nodes,* whereas all of the informants represented these nodes on the post-instruction task. However on the pre-instruction task *the intervening nodes* (nodes between the starting and finishing nodes) were only represented by two informants. Deficits in the representation of the intervening nodes resulted in the informants having difficulty: (a) tracking movements, and (b) implementing the solution structure. However on the post-instruction task, every informant represented all nodes. The *organisation of the nodes* for the solution structure was also of importance. On the pre-instruction task the solution structure was compromised when an informant: (a) did not identify the intervening nodes, (b) used an approximate rather than a precise position, or (c) drew the nodes as the solution strategy was being implemented. Although none of these responses was evident on the post-instruction task, two informants had difficulty with the solution structure because: (a) the spaces between the nodes were too small for accurate use, and (b) an error was made in calculating the quantity of the intervening nodes. Thus although difficulties were encountered with the organisation of the nodes on both the pre- and post-instruction tasks, there was an increase in the number of informants who were able to organise the nodes effectively after instruction as shown on Table 2.

Instruction appears to have had a positive effect on the representation of *nodes* because more students were able to: (a) represent the starting and finishing nodes, (b) represent the intervening nodes, and (c) organise the nodes for the implementation of the solution structure (see Table 2). Helen's diagrams in Figure 2 exemplify how informants typically depicted the starting and finishing nodes in *The Koala* and *The Frog* tasks by showing the base and top of the tree or well respectively. Although she has used some intervening nodes (the branches) in *The Koala* task, her organisation of these nodes did not facilitate the implementation of the solution structure. In contrast, she represented all intervening nodes on *The Frog* task and the organisation of these nodes facilitated the implementation of the solution structure. The second aspect of the theoretical prototype was *movement* which is now discussed.

Table 2

A Comparison of the Use of Nodes on the Pre- and Post-Instruction Tasks (n=12)

Node Description	Pre-instruction Task	Post-instruction Task
starting and finishing nodes accurately represented	8	12
intervening nodes accurately represented	2	12
organisation of nodes for the solution structure	2	10

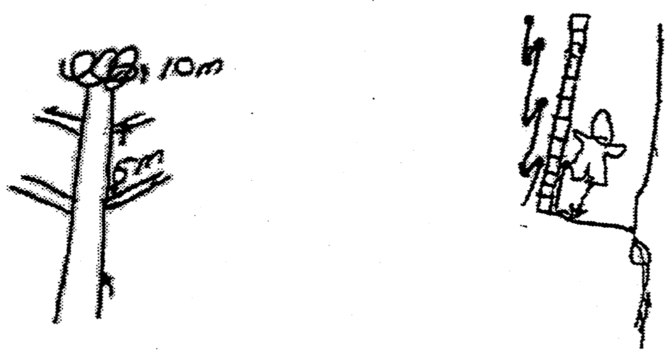


Figure 2. Helen's diagrams of *The Koala* and *The Frog* Tasks Respectively

On the pre-instruction task, *movement* was only represented by four informants, however all informants represented movement on the post-instruction task as shown on Table 3. Helen's diagrams (see Figure 2) exemplify the omission of drawn movement on the pre-instruction task, and the inclusion of drawn movement on the post-instruction task. However some of the informants who drew movement had difficulty with either the direction of the movement or the amount of movement (see Table 3). After instruction no informants had difficulty with the direction of movement, however six students were still unable to represent the amount of movement correctly. Four of the six students made the same error as Ben whose diagram is shown in Figure 3. On *The Koala* task Ben identified the base of the well as one brick high. Elise, whose error was the same as Ben's, made a similar error on *The Frog* task. The apparent increase in the number of informants experiencing difficulty with measurement on the post-instruction task is not interpreted as a negative effect of instruction, because, with the exception of Elise, none of the informants who had difficulty representing the amount of movement on the post-instruction task attempted to represent movement on the pre-instruction task. The instructional program is interpreted to have had a positive effect on the generation of diagrams because there were increases in the number of informants, as shown on Table 3, who: (a) represented movement, (b) correctly represented directionality, and (c) correctly represented the amount of movement.

Table 3
A Comparison of Informants' Representation of Movement in the Pre- and Post-Instruction Interviews (*n*=12)

	The Koala task	The Frog task
the use of a representation to depict movement	4	12
direction of the movement represented correctly	3	12
amount of movement represented correctly	1	6

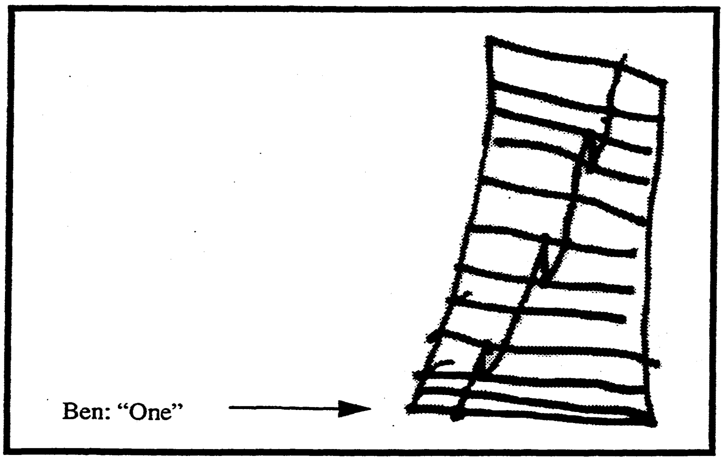
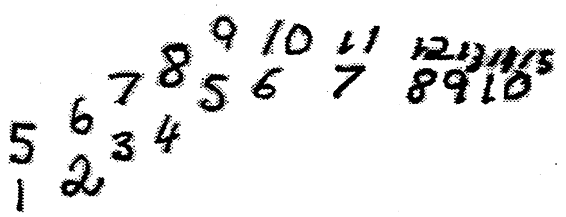


Figure 3. Ben's Error in the Representation of Movement on *The Frog* Task

There was also an improvement in the number of informants who represented both the movement correctly and the nodes correctly. Whereas no informants had a totally accurate representation prior to instruction, five informants' representations were totally accurate after instruction. Hence Assertion 2 is supported because there was an increase in the number of informants who represented: (a) the correct type of diagram, (b) either the nodes or movement accurately on the diagram, and (c) the nodes and movement accurately on the diagram.

The employment of a theoretical prototype was particularly useful for ascertaining the qualitative changes in an informants' diagrams. For example, Kate's diagrams on the pre- and post-instruction tasks, as shown on Figure 4, both represent all aspects of movement and the starting node, however on the pre-instruction task there are deficits in her representation of the intervening nodes, the ending node, and the organisation of the nodes, all of which were overcome on the post-instruction task. The contrast between Kate's elegant post-instruction diagram, as shown on Figure 4 and Helen's pre-instruction diagram (see Figure 2) exemplifies the expert-novice difference, in that the expert's diagram (Kate) is a structural representation, whereas the novice's diagram (Helen) is a literal representation.



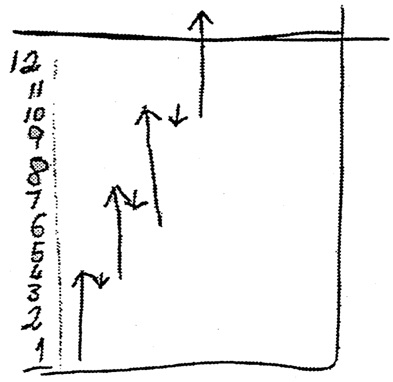


Figure 4. Kate's Diagrams for *The Koala* and *The Frog* Tasks Respectively.

Implications

The assumption of this study is that diagram generation is important because diagrams facilitate the conceptualisation of a problem (van Essen & Hamaker, 1990). The results of this study have five implications. Firstly, this study provides evidence that an instructional program can improve students' generation of diagrams. Secondly, specific quantitative and qualitative changes in diagrams can be identified using a theoretical framework to establish appropriate evaluation criteria. Thirdly, the success of the instructional program can be attributed to the theoretical framework that supports the role of the generation of diagrams in problem solving. Fourthly, quantitative changes should not be under-rated because they provide evidence that a student is becoming familiar with the domain of diagrams as a unique representational system. Representational systems are problem spaces in their own right *prior* to becoming useful "cognitive" tools (Landsman & Karmiloff-Smith, 1992). Finally, an improvement in students' generation of diagrams will require an informed and proactive stance by teachers and teacher educators, who may need to consciously re-educate themselves to include visual literacy in mathematics instruction.

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